MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 9 · Further induction III: induction in context

Name: _____ Date: _____

Recursive sequences

1. A sequence has $a_1 = 3$ and $a_{n+1} = 2a_n - 1$. Prove by induction that $a_n = 2^n + 1$ for all integers $n \geq 1$.

2. A sequence has $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$. Prove by induction that $u_n = n^2$ for all integers $n \geq 1$.

Calculus

3. Prove by induction that $\dfrac{d^n}{dx^n}\left(xe^{-x}\right) = (-1)^n(x-n)e^{-x}$ for all integers $n \geq 1$.

Binomial theorem and logarithms

4. Use Pascal's identity to prove by induction that $\displaystyle\sum_{r=0}^{n}\binom{n}{r} = 2^n$ for all integers $n \geq 1$.

5. Prove by induction that $\displaystyle\sum_{r=2}^{n}\ln\left(1-\frac{1}{r}\right) = -\ln n$ for all integers $n \geq 2$.

Geometry

6. Prove by induction that the interior angles of a convex polygon with n sides sum to $(n-2)\times 180^\circ$, for all integers $n \geq 3$.

Adapted HSC-style

7. A sequence is defined by $a_1 = 1$ and $a_{n+1} = \sqrt{2a_n + 3}$ for $n \geq 1$. (a) Prove by induction that $0 < a_n < 3$ for all integers $n \geq 1$. (b) Prove by induction that $a_{n+1} > a_n$ for all integers $n \geq 1$.