MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 8 · Further induction II: inequalities

Name: _____ Date: _____

Choosing the first value

1. Find the smallest positive integer n from which each inequality is true for all larger n as well, by testing small values.

(a) $2^n > 2n + 1$

(b) $n! > n^2$

(c) $2^n > n^2$

Induction with inequalities

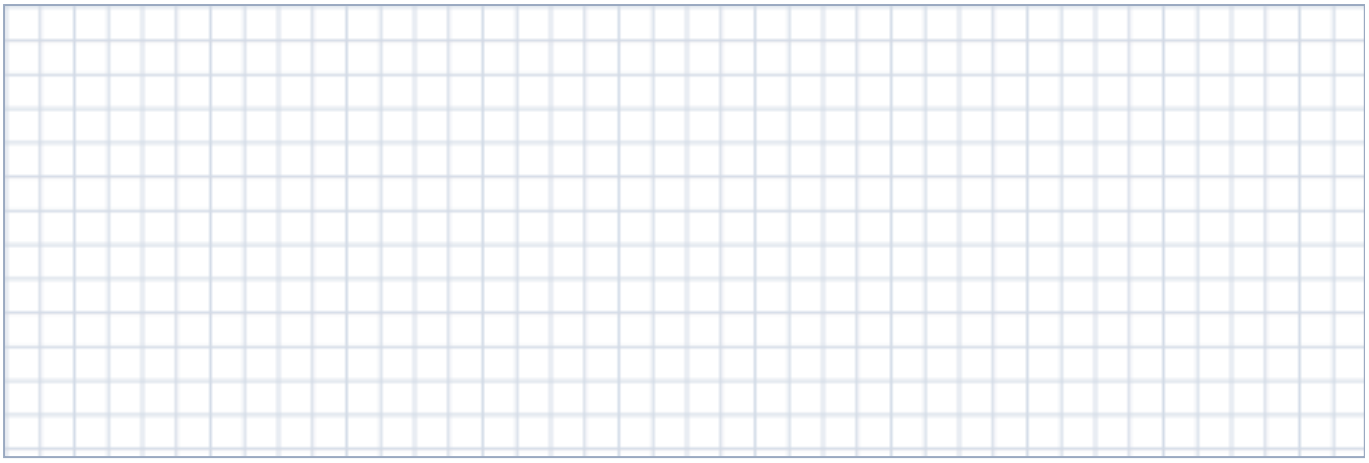
2. Prove by induction that $2^n > 2n + 1$ for all integers $n \geq 3$.

3. Prove by induction that $4^n \geq 1 + 3n$ for all integers $n \geq 1$.

4. Prove by induction that $n! \geq 2^{n-1}$ for all integers $n \geq 1$.

5. Prove by induction that $(1 + x)^n \geq 1 + nx$ for all integers $n \geq 1$, where $x > -1$ is a fixed real number.

6. Prove by induction that $\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\geq\frac{1}{2}$ for all integers $n\geq 1$.



Adapted HSC-style

7. (a) Show that $2k^3-3k^2-3k-1>0$ for integers $k\geq 4$. (b) Hence prove by induction that $3^n>n^3$ for all integers $n\geq 4$.

