MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 7 · Further induction I: series and divisibility

Name: _____ Date: _____

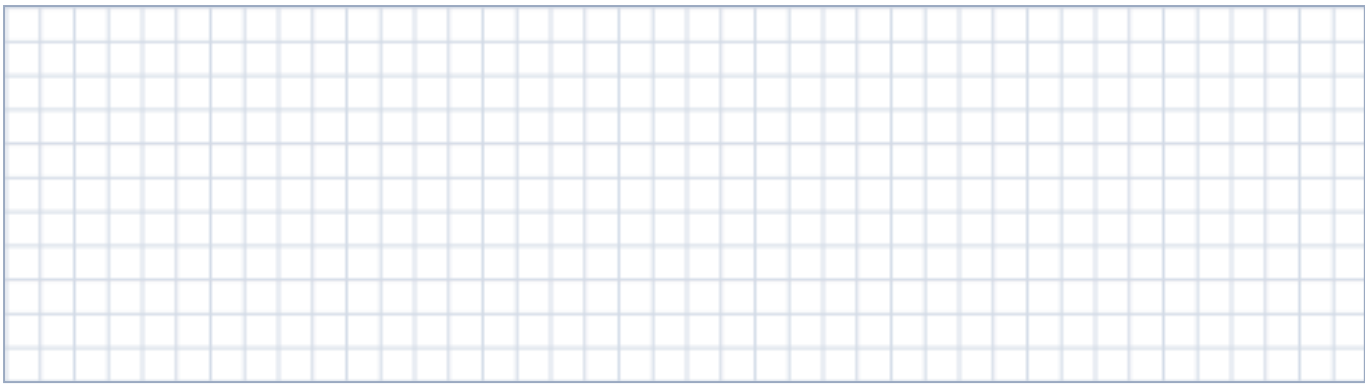
Series

1. Prove by induction that for all integers $n \geq 1$, $1 \times 3 + 2 \times 4 + \cdots + n(n+2) = \dfrac{n(n+1)(2n+7)}{6}$.

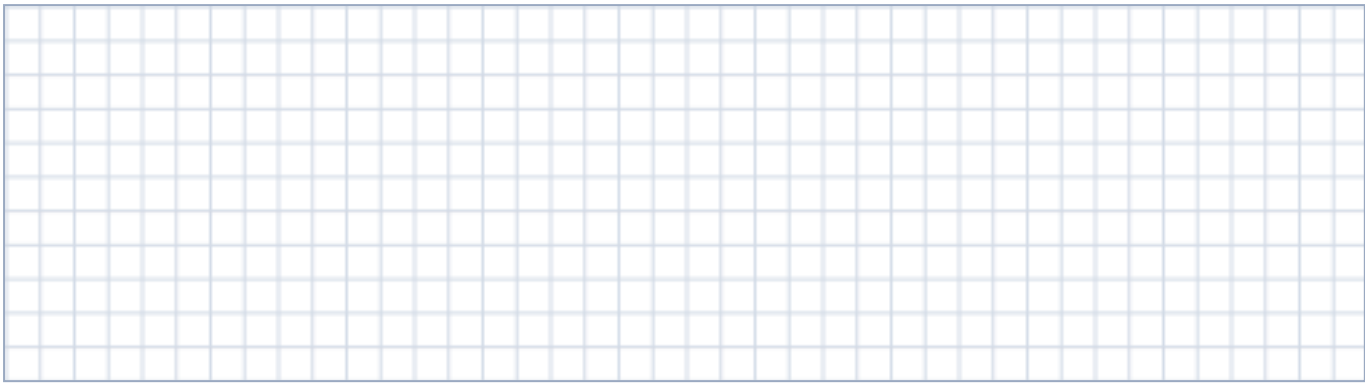
2. Prove by induction that for all integers $n \geq 1$, $\displaystyle\sum_{r=1}^{n} r \cdot 2^r = (n-1)2^{n+1} + 2$.

Divisibility

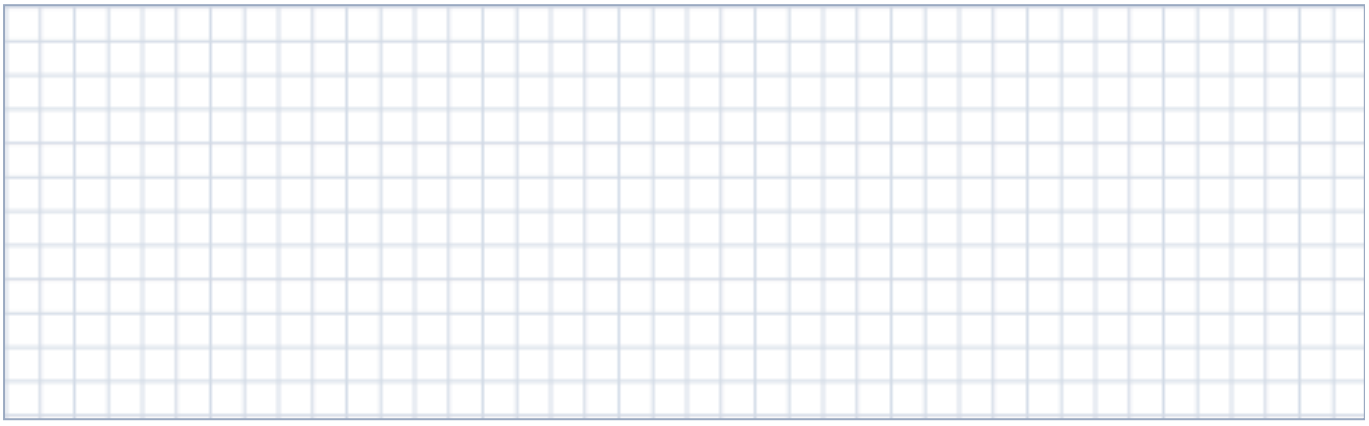
3. Prove by induction that $9^n + 7$ is divisible by 8 for all integers $n \geq 1$.



4. Prove by induction that $5^n - 4n - 1$ is divisible by 16 for all integers $n \geq 1$.



5. Prove by induction that $3^n + 5^n$ is divisible by 8 for all odd positive integers n.



Adapted HSC-style

6. (a) Prove by induction that for all integers $n \geq 1$, $\displaystyle\sum_{r=1}^{n} \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$. [3] (b) Hence find $\displaystyle\sum_{r=1}^{9} \frac{r}{(r+1)!}$ as a fraction. [1]

