MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 6 · Algebraic inequalities II

Name: _____ Date: _____

AM–GM

1. Prove that $x + \dfrac{16}{x} \geq 8$ for $x > 0$, and state when equality holds.

2. Prove that $(a+1)(b+1)(a+b) \geq 8ab$ for $a, b > 0$.

3. Prove that $|a - b| \leq |a| + |b|$ for real a, b.

4. Prove that $\big||a|-|b|\big| \le |a-b|$. (Hint: use $|a| = |(a-b)+b|$ and $|b| = |(b-a)+a|$.)

5. Two sides of a triangle are 7 cm and 12 cm. (a) Find the possible lengths of the third side. (b) How many integer lengths are possible? (c) If the triangle is isosceles, find its sides.

Geometry

6. A rectangle has area $36\ \text{cm}^2$. Prove that its perimeter is at least 24 cm, and state when it equals 24 cm.

7. A right-angled triangle has hypotenuse 10 cm. Prove that its perimeter is at most $10+10\sqrt{2}$ cm. When is this achieved?

Adapted HSC-style

8. (a) Prove that $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right) \geq 4$ for $a, b > 0$. [2] (b) Hence show that if $a+b=1$, then $\frac{1}{a}+\frac{1}{b} \geq 4$. [1] (c) Hence show that if $a, b > 0$ and $a+b=1$, then $\frac{1}{a^2}+\frac{1}{b^2} \geq 8$. [2]

