MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 5 · Algebraic inequalities I

Name: _____ Date: _____

Squares are non-negative

1. Prove each inequality for all real numbers.

(a) $x^2 - 6x + 10 > 0$ (b) $a^2 + b^2 + 2 \geq 2(a + b)$

(c) $x^2 + y^2 + 1 \geq x + y + xy$

2. Prove that $a^4 + b^4 \geq a^3 b + ab^3$ for all real a and b.

Properties of inequalities

3. Give a counterexample to each statement.

(a) If $a > b$ and $c > d$, then $a - c > b - d$.

(b) If $a > b$ and $c > d$, then $\frac{a}{c} > \frac{b}{d}$.

4. Given $a > b > 0$, prove that $\frac{a}{a+1} > \frac{b}{b+1}$.

Using a given condition

5. Let $a, b > 0$ with $a + b = 2$. (a) Prove that $ab \leq 1$. (b) Prove that $a^2 + b^2 \geq 2$. (c) Prove that $\frac{1}{a} + \frac{1}{b} \geq 2$.

6. For $a, b, c > 0$, prove that $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$.

Spot the flaw

7. A student "proves" that $x^2 + 1 \geq 2x$ as follows: "$x^2 + 1 \geq 2x \Rightarrow x^2 - 2x + 1 \geq 0 \Rightarrow (x-1)^2 \geq 0$, which is true, so the claim holds." Explain the flaw and write a correct proof.

Adapted HSC-style

8. (a) Prove that $x^2 + y^2 \geq 2xy$ for all real x, y. [1] (b) Hence show that $(a+b)^2 \leq 2(a^2+b^2)$ for all real a, b. [1] (c) Hence show that if $a + b + c = 1$, then $a^2 + b^2 + c^2 \geq \frac{1}{3}$. [2]