MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 3 · Number proofs

Name: _____ Date: _____

Direct proofs

1. Prove that the product of an even integer and any integer is even.

2. Prove that the sum of the squares of two consecutive integers is odd.

3. Let $3 \mid a$ and $3 \mid b$. Prove that $3 \mid (a^2 + 5b)$.

Cases

4. By considering the cases n even and n odd, prove that the square of any integer has the form $4m$ or $4m + 1$.

Disproving claims and inequalities

5. A student claims: "If a divides bc, then a divides b or a divides c." Disprove the claim with a counterexample, and say for which numbers a the claim would be true.

6. Prove that $(3+4)^n \neq 3^n + 4^n$ for all integers $n \geq 2$.

HSC-style

7. Adapted HSC-style.
(i) Show that $n^3 + 3n^2 + 2n = n(n+1)(n+2)$. **(1 mark)**
(ii) Prove that $n^3 + 3n^2 + 2n$ is divisible by 6 for all integers n. **(2 marks)**
(iii) Disprove: "$n^3 + 3n^2 + 2n$ is divisible by 12 for all integers n." **(1 mark)**