MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 10 · Calculus and the squeeze theorem

Name: _____ Date: _____

Proving inequalities

1. Prove that $\ln x \leq x - 1$ for all $x > 0$.

2. Prove that $e^x > 1 + x + \dfrac{x^2}{2}$ for all $x > 0$.

3. Using $\sin x > x - \dfrac{x^3}{6}$ for $x > 0$ (the exit ticket result), prove that $\cos x \leq 1 - \dfrac{x^2}{2} + \dfrac{x^4}{24}$ for all real x.

4. Show that $f(x) = \dfrac{\ln x}{x}$ is decreasing for $x > e$. Hence show that $5^6 > 6^5$.

The squeeze theorem

5. Use the squeeze theorem to find:

(a) $\lim\limits_{x \to 0} x \cos\left(\dfrac{1}{x}\right)$ (b) $\lim\limits_{x \to \infty} \dfrac{2 + \cos x}{x}$

6. Use $\cos x \geq 1 - \dfrac{x^2}{2}$ to show that $0 \leq 1 - \cos x \leq \dfrac{x^2}{2}$ for all real x. Hence find $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x}$.

Adapted HSC-style

7. (a) Show that $\ln(1+x) < x$ for $x > 0$. (b) Hence show that $\left(1+\frac{1}{n}\right)^n < e$ for all positive integers n.

(c) Given that $\ln(1+x) > \frac{x}{1+x}$ for $x > 0$, show that $\left(1+\frac{1}{n}\right)^{n+1} > e$.

