MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 1 · Logic and the language of proof

Name: _____ Date: _____

Statements

1. Decide which of these are statements. For each statement, say whether it is true or false.

(a) 7 is a prime number.

(b) How old are you?

(c) $2x + 3 = 11$

(d) Every square is a rectangle.

2. Let P: "n is divisible by 4" and Q: "n is even". For $n = 6$, state whether each is true or false: (a) $P \wedge Q$ (b) $P \vee Q$ (c) $\neg P$ (d) $P \Rightarrow Q$.

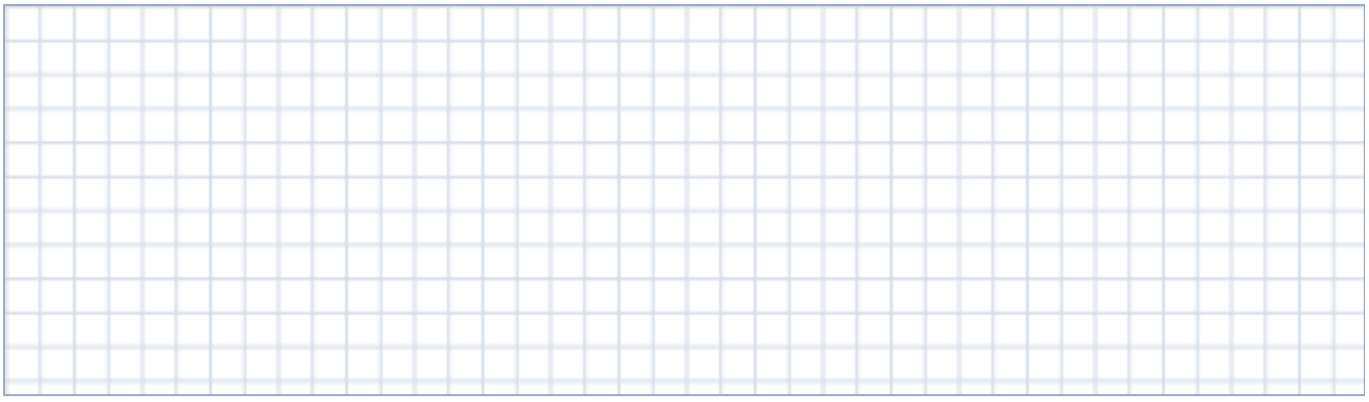
Negation

3. Write the negation of each statement.

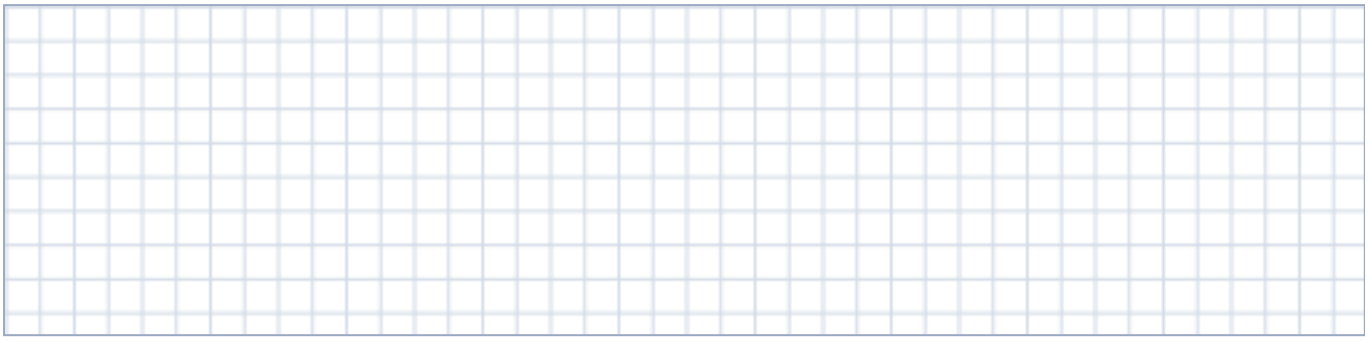
(a) All triangles have a right angle. (b) $x \geq 3$

(c) n is prime and n is odd. (d) a is positive or b is negative.

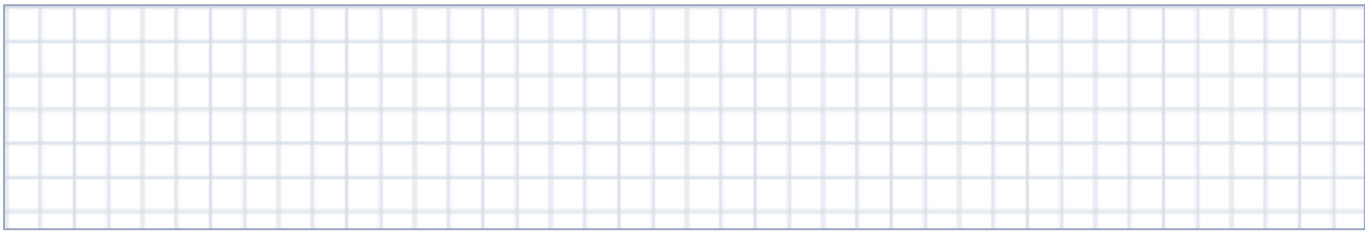
(e) If $x > 0$, then $x^2 > 0$.



4. Complete a truth table (using 1 and 0) to show that $\neg(P \wedge Q) \simeq \neg P \vee \neg Q$.



5. Use $P \Rightarrow Q \simeq \neg P \vee Q$ to show that $\neg(P \Rightarrow Q) \simeq P \wedge \neg Q$.



Implication, exhaustion and counterexamples

6. Let P: x is a multiple of 4 and Q: x is a multiple of 8. Find a value of x for each row of the truth table for $P \Rightarrow Q$ (rows P, Q = 1,1; 1,0; 0,1; 0,0). Is $P \Rightarrow Q$ always true?

7. (a) Prove by exhaustion: all prime numbers between 20 and 30 are odd. (b) Use a counterexample to show that the statement "For all real x, $x^2 > x$" is false.

Adapted HSC-style

8. **Adapted HSC-style.** Consider: "If n is a multiple of 4 and a multiple of 6, then n is a multiple of 24." (a) Write the negation. (b) Is the original statement true? Justify your answer.