MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 7 · Complex numbers as vectors

Name: _____ Date: _____

Vectors and parallelograms

1. P, Q and R represent $1+4i$, $-2+i$ and $3-2i$. Find the complex numbers represented by (a) $\overrightarrow{PQ}$, (b) $\overrightarrow{QR}$, (c) $\overrightarrow{RP}$, and show that $\overrightarrow{PQ}+\overrightarrow{QR}+\overrightarrow{RP}=0$.

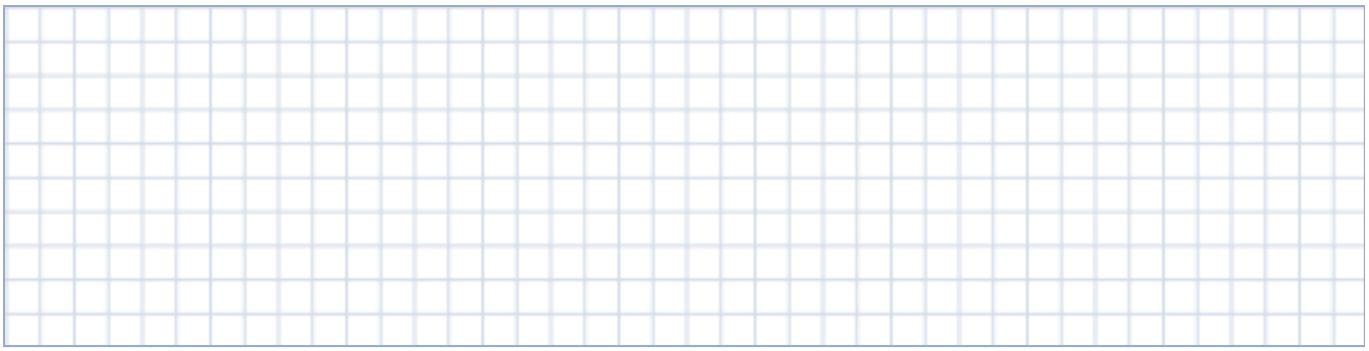
2. $OABC$ is a parallelogram with A at $6-i$ and C at $-1+4i$. Find B, $\overrightarrow{AC}$ and the lengths of the two diagonals.

Distance and direction

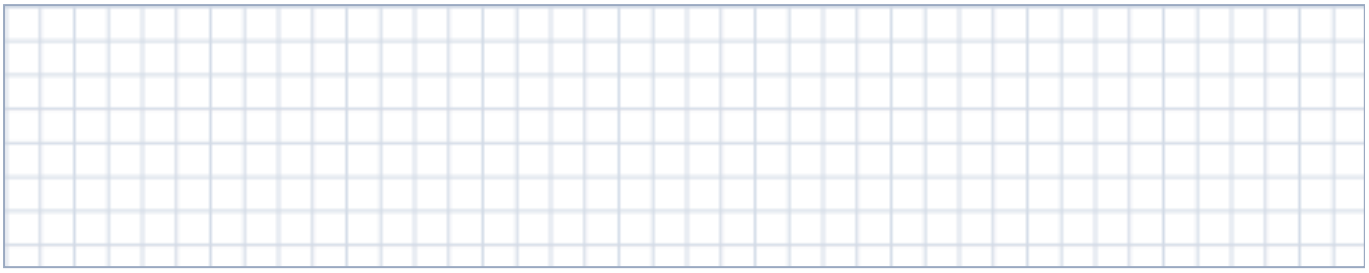
3. W is $2+3i$ and Z is $5+7i$. Find $|z-w|$ and sketch $\overrightarrow{WZ}$ on an Argand diagram. Find $\arg(z-w)$ correct to two decimal places.

Triangle inequality

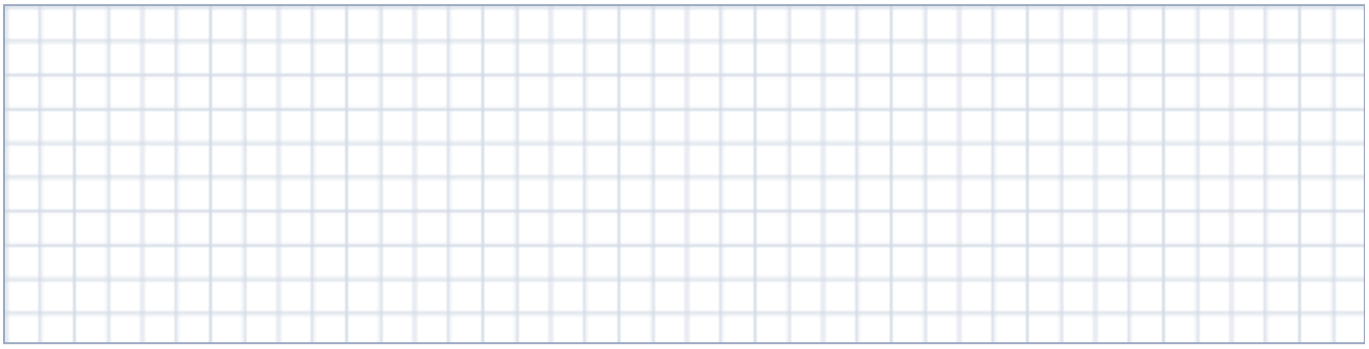
4. Show that for any complex numbers $|z_1 - z_2| \geq \big||z_1| - |z_2|\big|$.



5. Given $|z| \leq 2$, find the greatest possible value of $|z^2 + 3z|$ using the triangle inequality, and explain why it is actually attained.



6. Prove that $|z + w|^2 + |z - w|^2 = 2|z|^2 + 2|w|^2$, and interpret it for a parallelogram.



HSC-style

7. Adapted HSC-style. Let $z = \operatorname{cis}\dfrac{\pi}{3}$ and $w = \operatorname{cis}\dfrac{\pi}{2}$. (a) Show that $|z+w| = 2\cos\dfrac{\pi}{12}$. (b) Hence find $\arg(z+w)$ and explain your answer using a rhombus. (c) By equating real parts, show that $2\cos\dfrac{\pi}{12}\cos\dfrac{5\pi}{12} = \dfrac{1}{2}$.

