MCINTYRE MATHS · MATHEMATICS EXTENSION 2

Practice worksheet

Lesson 6 · Multiplying and dividing in polar form

Name: _____ Date: _____

Multiplying and dividing

1. Express in polar form with the principal argument.

(a) $3\operatorname{cis}\frac{2\pi}{3}\times 2\operatorname{cis}\frac{3\pi}{4}$ (b) $\dfrac{8\operatorname{cis}\frac{\pi}{6}}{2\operatorname{cis}\frac{5\pi}{6}}$

(c) $\left(\sqrt{2}\operatorname{cis}\frac{3\pi}{4}\right)^3$

2. Write $z_1=\sqrt{3}-i$ and $z_2=-1+i$ in polar form, then find z_1z_2 and $\dfrac{z_1}{z_2}$ in polar form.

Exact values

3. Let $z_1 = \sqrt{3} + i$ and $z_2 = 1 - i$. Find $z_1 z_2$ in polar form and in Cartesian form. Hence find the exact value of $\cos\left(-\frac{\pi}{12}\right)$.

Proofs

4. Prove that $\arg\left(\frac{1}{z}\right) = -\arg z$ for $z \neq 0$, and explain the connection with the conjugate $\overline{z}$.

5. Prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ using polar form.

Powers

6. Use polar form to find $(1-i)^8$ and $\left(-1+\sqrt{3}\,i\right)^6$.

HSC-style

7. Let $z=1+i$ and $w=\sqrt{3}+i$. (a) Write z and w in polar form. (b) Find $\dfrac{w^6}{z^4}$. (c) Find the smallest positive integer n such that $\left(\dfrac{w}{z}\right)^n$ is real.